

Tema 6: Funciones recursivas

Informática (2019–20)

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Tema 6: Funciones recursivas

1. Recursión numérica
2. Recusión sobre lista
3. Recursión sobre varios argumentos
4. Recursión múltiple
5. Recursión mutua
6. Heurísticas para las definiciones recursivas

Recursión numérica: El factorial

- ▶ La función factorial:

```
factorial :: Integer -> Integer
factorial 0 = 1
factorial n = n * factorial (n-1)
```

- ▶ Cálculo:

```
factorial 3 = 3 * (factorial 2)
            = 3 * (2 * (factorial 1))
            = 3 * (2 * (1 * (factorial 0)))
            = 3 * (2 * (1 * 1))
            = 3 * (2 * 1)
            = 3 * 2
            = 6
```

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            = 3 * (2 * 1)
            = 3 * 2
            = 6
```

Recursión numérica: El producto

- ▶ Definición recursiva del producto:

```
por :: Int -> Int -> Int
m 'por' 0 = 0
m 'por' n = m + (m 'por' (n-1))
```

- ▶ Cálculo:

```
3 'por' 2 = 3 + (3 'por' 1)
           = 3 + (3 + (3 'por' 0))
           = 3 + (3 + 0)
           = 3 + 3
           = 6
```

Recursión numérica: El producto

- Definición recursiva del producto:

```
por :: Int -> Int -> Int
m 'por' 0 = 0
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           = 3 + (3 + 0)
           = 3 + 3
           = 6
```

Recursión numérica: El producto

- Definición recursiva del producto:

```
por :: Int -> Int -> Int
m 'por' 0 = 0
m 'por' n = m + (m 'por' (n-1))
```

- Cálculo:

$$\begin{aligned} 3 \text{ 'por' } 2 &= 3 + (3 \text{ 'por' } 1) \\ &= 3 + (3 + (3 \text{ 'por' } 0)) \\ &= 3 + (3 + 0) \\ &= 3 + 3 \\ &= 6 \end{aligned}$$

Recursión sobre listas: La función product

- ▶ Producto de una lista de números:

```
----- Prelude -----  
product :: Num a => [a] -> a  
product []      = 1  
product (n:ns) = n * product ns
```

- ▶ Cálculo:

```
product [7,5,2] = 7 * (product [5,2])  
                = 7 * (5 * (product [2]))  
                = 7 * (5 * (2 * (product [])))  
                = 7 * (5 * (2 * 1))  
                = 7 * (5 * 2)  
                = 7 * 10  
                = 70
```

Recursión sobre listas: La función product

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_____ Prelude _____  
product :: Num a => [a] -> a  
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```

Recursión sobre listas: La función product

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Prelude

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product :: Num a => [a] -> a
product []      = 1
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- ▶ Cálculo:

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product [7,5,2] = 7 * (product [5,2])
                = 7 * (5 * (product [2]))
                = 7 * (5 * (2 * (product [])))
                = 7 * (5 * (2 * 1))
                = 7 * (5 * 2)
                = 7 * 10
                = 70
```

Recursión sobre listas: La función length

- ▶ Longitud de una lista:

```
----- Prelude -----  
length :: [a] -> Int  
length []      = 0  
length (_:xs) = 1 + length xs
```

- ▶ Cálculo:

```
length [2,3,5] = 1 + (length [3,5])  
               = 1 + (1 + (length [5]))  
               = 1 + (1 + (1 + (length [])))  
               = 1 + (1 + (1 + 0))  
               = 1 + (1 + 1)  
               = 1 + 2  
               = 3
```

Recursión sobre listas: La función length

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```
length :: [a] -> Int
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length [] = 0
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length [2,3,5] = 1 + (length [3,5])  
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Recursión sobre listas: La función length

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```
length [2,3,5] = 1 + (length [3,5])
               = 1 + (1 + (length [5]))
               = 1 + (1 + (1 + (length [])))
               = 1 + (1 + (1 + 0))
               = 1 + (1 + 1)
               = 1 + 2
               = 3
```

Recursión sobre listas: La función reverse

► Inversa de una lista:

```
_____ Prelude _____  
reverse :: [a] -> [a]  
reverse []      = []  
reverse (x:xs) = reverse xs ++ [x]
```

► Cálculo:

```
reverse [2,5,3] = (reverse [5,3]) ++ [2]  
                = ((reverse [3]) ++ [5]) ++ [2]  
                = (((reverse []) ++ [3]) ++ [5]) ++ [2]  
                = (([] ++ [3]) ++ [5]) ++ [2]  
                = ([3] ++ [5]) ++ [2]  
                = [3,5] ++ [2]  
                = [3,5,2]
```

Recursión sobre listas: La función reverse

- Inversa de una lista:

Prelude

```
reverse :: [a] -> [a]
reverse []      = []
reverse (x:xs) = reverse xs ++ [x]
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- Cálculo:

```
reverse [2,5,3] = (reverse [5,3]) ++ [2]
                = ((reverse [3]) ++ [5]) ++ [2]
                = (((reverse []) ++ [3]) ++ [5]) ++ [2]
                = (([] ++ [3]) ++ [5]) ++ [2]
                = ([3] ++ [5]) ++ [2]
                = [3,5] ++ [2]
                = [3,5,2]
```


Recursión sobre listas: La función reverse

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reverse :: [a] -> [a]
reverse []      = []
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```

- Cálculo:

```
reverse [2,5,3] = (reverse [5,3]) ++ [2]
                = ((reverse [3]) ++ [5]) ++ [2]
                = (((reverse []) ++ [3]) ++ [5]) ++ [2]
                = (([] ++ [3]) ++ [5]) ++ [2]
                = ([3] ++ [5]) ++ [2]
                = [3,5] ++ [2]
                = [3,5,2]
```

Recursión sobre listas: ++

► Concatenación de listas:

Prelude

```
(++) :: [a] -> [a] -> [a]
[]      ++ ys = ys
(x:xs) ++ ys = x : (xs ++ ys)
```

► Cálculo:

```
[1,3,5] ++ [2,4] = 1:([3,5] ++ [2,4])
                  = 1:(3:([5] ++ [2,4]))
                  = 1:(3:(5:([ ] ++ [2,4])))
                  = 1:(3:(5:[2,4]))
                  = 1:(3:[5,2,4])
                  = 1:[3,5,2,4]
                  = [1,3,5,2,4]
```

Recursión sobre listas: ++

- Concatenación de listas:

Prelude

```
(++) :: [a] -> [a] -> [a]
[]    ++ ys = ys
(x:xs) ++ ys = x : (xs ++ ys)
```

- Cálculo:

```
[1,3,5] ++ [2,4] = 1:([3,5] ++ [2,4])
                 = 1:(3:([5] ++ [2,4]))
                 = 1:(3:(5:([] ++ [2,4])))
                 = 1:(3:(5:[2,4]))
                 = 1:(3:[5,2,4])
                 = 1:[3,5,2,4]
                 = [1,3,5,2,4]
```

Recursión sobre listas: ++

- Concatenación de listas:

Prelude

```
(++) :: [a] -> [a] -> [a]
[]    ++ ys = ys
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- Cálculo:

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[1,3,5] ++ [2,4] = 1:([3,5] ++ [2,4])
                 = 1:(3:([5] ++ [2,4]))
                 = 1:(3:(5:([] ++ [2,4])))
                 = 1:(3:(5:[2,4]))
                 = 1:(3:[5,2,4])
                 = 1:[3,5,2,4]
                 = [1,3,5,2,4]
```

Recursión sobre listas: Inserción ordenada

- ▶ (`inserta e xs`) inserta el elemento `e` en la lista `xs` delante del primer elemento de `xs` mayor o igual que `e`. Por ejemplo,

```
inserta 5 [2,4,7,3,6,8,10] ~> [2,4,5,7,3,6,8,10]
```

```
inserta :: Ord a => a -> [a] -> [a]
inserta e []                = [e]
inserta e (x:xs) | e <= x  = e : (x:xs)
                  | otherwise = x : inserta e xs
```

- ▶ Cálculo:

```
inserta 4 [1,3,5,7] = 1:(inserta 4 [3,5,7])
                   = 1:(3:(inserta 4 [5,7]))
                   = 1:(3:(4:(5:[7])))
                   = 1:(3:(4:[5,7]))
                   = [1,3,4,5,7]
```

Recursión sobre listas: Inserción ordenada

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```

inserta :: Ord a => a -> [a] -> [a]
inserta e []                = [e]
inserta e (x:xs) | e <= x  = e : (x:xs)
                   | otherwise = x : inserta e xs

```

- ▶ Cálculo:

```

inserta 4 [1,3,5,7] = 1:(inserta 4 [3,5,7])
                   = 1:(3:(inserta 4 [5,7]))
                   = 1:(3:(4:(5:[7])))
                   = 1:(3:(4:[5,7]))
                   = [1,3,4,5,7]

```

Recursión sobre listas: Inserción ordenada

- ▶ (`inserta e xs`) inserta el elemento `e` en la lista `xs` delante del primer elemento de `xs` mayor o igual que `e`. Por ejemplo,

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inserta 5 [2,4,7,3,6,8,10] ~> [2,4,5,7,3,6,8,10]
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```
inserta :: Ord a => a -> [a] -> [a]
inserta e []                = [e]
inserta e (x:xs) | e <= x  = e : (x:xs)
                  | otherwise = x : inserta e xs
```

- ▶ Cálculo:

```
inserta 4 [1,3,5,7] = 1:(inserta 4 [3,5,7])
                   = 1:(3:(inserta 4 [5,7]))
                   = 1:(3:(4:(5:[7])))
                   = 1:(3:(4:[5,7]))
                   = [1,3,4,5,7]
```

Recursión sobre listas: Ordenación por inserción

- ▶ (`ordena_por_insercion xs`) es la lista `xs` ordenada mediante inserción, Por ejemplo,

```
|ordena_por_insercion [2,4,3,6,3] ~> [2,3,3,4,6]
```

```
ordena_por_insercion :: Ord a => [a] -> [a]
```

```
ordena_por_insercion [] = []
```

```
ordena_por_insercion (x:xs) =
```

```
    inserta x (ordena_por_insercion xs)
```

- ▶ Cálculo:

```
ordena_por_insercion [7,9,6] =
= inserta 7 (inserta 9 (inserta 6 []))
= inserta 7 (inserta 9 [6])
= inserta 7 [6,9]
= [6,7,9]
```


Recursión sobre listas: Ordenación por inserción

- ▶ `(ordena_por_insercion xs)` es la lista `xs` ordenada mediante inserción, Por ejemplo,

```
|ordena_por_insercion [2,4,3,6,3] ~> [2,3,3,4,6]
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```
ordena_por_insercion :: Ord a => [a] -> [a]
```

```
ordena_por_insercion [] = []
```

```
ordena_por_insercion (x:xs) =
```

```
  inserta x (ordena_por_insercion xs)
```

- ▶ Cálculo:

```
ordena_por_insercion [7,9,6] =
= inserta 7 (inserta 9 (inserta 6 []))
= inserta 7 (inserta 9 [6])
= inserta 7 [6,9]
= [6,7,9]
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- ▶ Cálculo:

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|ordena_por_insercion [7,9,6] =
= inserta 7 (inserta 9 (inserta 6 []))
= inserta 7 (inserta 9 [6])
= inserta 7 [6,9]
= [6,7,9]
```

Recursión sobre varios argumentos: La función zip

- ▶ Emparejamiento de elementos (la función zip):

```

Prelude
zip :: [a] -> [b] -> [(a, b)]
zip [] _          = []
zip _ []         = []
zip (x:xs) (y:ys) = (x,y) : zip xs ys

```

- ▶ Cálculo:

```

zip [1,3,5] [2,4,6,8]
= (1,2) : (zip [3,5] [4,6,8])
= (1,2) : ((3,4) : (zip [5] [6,8]))
= (1,2) : ((3,4) : ((5,6) : (zip [] [8])))
= (1,2) : ((3,4) : ((5,6) : []))
= [(1,2), (3,4), (5,6)]

```

Recursión sobre varios argumentos: La función zip

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Prelude
zip :: [a] -> [b] -> [(a, b)]
zip [] _          = []
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zip (x:xs) (y:ys) = (x,y) : zip xs ys
  
```

- ▶ Cálculo:

```

zip [1,3,5] [2,4,6,8]
= (1,2) : (zip [3,5] [4,6,8])
= (1,2) : ((3,4) : (zip [5] [6,8]))
= (1,2) : ((3,4) : ((5,6) : (zip [] [8])))
= (1,2) : ((3,4) : ((5,6) : []))
= [(1,2), (3,4), (5,6)]
  
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zip _ []         = []
zip (x:xs) (y:ys) = (x,y) : zip xs ys

```

- ▶ Cálculo:

```

zip [1,3,5] [2,4,6,8]
= (1,2) : (zip [3,5] [4,6,8])
= (1,2) : ((3,4) : (zip [5] [6,8]))
= (1,2) : ((3,4) : ((5,6) : (zip [] [8])))
= (1,2) : ((3,4) : ((5,6) : []))
= [(1,2), (3,4), (5,6)]

```

Recursión sobre varios argumentos: La función drop

- Eliminación de elementos iniciales:

Prelude

```
drop :: Int -> [a] -> [a]
drop 0 xs      = xs
drop n []      = []
drop n (x:xs) = drop (n-1) xs
```

- Cálculo:

drop 2 [5,7,9,4]		drop 5 [1,4]
= drop 1 [7,9,4]		= drop 4 [4]
= drop 0 [9,4]		= drop 1 []
= [9,4]		= []

Recursión sobre varios argumentos: La función drop

- Eliminación de elementos iniciales:

Prelude

```
drop :: Int -> [a] -> [a]
drop 0 xs      = xs
drop n []      = []
drop n (x:xs) = drop (n-1) xs
```

- Cálculo:

drop 2 [5,7,9,4]		drop 5 [1,4]
= drop 1 [7,9,4]		= drop 4 [4]
= drop 0 [9,4]		= drop 1 []
= [9,4]		= []

Recursión sobre varios argumentos: La función drop

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Prelude

```
drop :: Int -> [a] -> [a]
drop 0 xs      = xs
drop n []      = []
drop n (x:xs) = drop (n-1) xs
```

- Cálculo:

drop 2 [5,7,9,4]		drop 5 [1,4]
= drop 1 [7,9,4]		= drop 4 [4]
= drop 0 [9,4]		= drop 1 []
= [9,4]		= []

Recursión múltiple: La función de Fibonacci

- ▶ La sucesión de Fibonacci es: 0,1,1,2,3,5,8,13,21,... Sus dos primeros términos son 0 y 1 y los restantes se obtienen sumando los dos anteriores.
- ▶ `(fibonacci n)` es el n -ésimo término de la sucesión de Fibonacci. Por ejemplo,

```
| fibonacci 8 ~> 21
```

```
fibonacci :: Int -> Int
fibonacci 0 = 0
fibonacci 1 = 1
fibonacci n = fibonacci (n-2) + fibonacci (n-1)
```

Recursión múltiple: La función de Fibonacci

- ▶ La sucesión de Fibonacci es: 0,1,1,2,3,5,8,13,21,... Sus dos primeros términos son 0 y 1 y los restantes se obtienen sumando los dos anteriores.
- ▶ (`fibonacci n`) es el n -ésimo término de la sucesión de Fibonacci. Por ejemplo,

| `fibonacci 8` \rightsquigarrow 21

```
fibonacci :: Int -> Int
```

```
fibonacci 0 = 0
```

```
fibonacci 1 = 1
```

```
fibonacci n = fibonacci (n-2) + fibonacci (n-1)
```

Recursión múltiple: Ordenación rápida

- ▶ Algoritmo de ordenación rápida:

```
ordena :: (Ord a) => [a] -> [a]
ordena [] = []
ordena (x:xs) =
    (ordena menores) ++ [x] ++ (ordena mayores)
  where menores = [a | a <- xs, a <= x]
        mayores = [b | b <- xs, b > x]
```

Recursión múltiple: Ordenación rápida

- ▶ Algoritmo de ordenación rápida:

```
ordena :: (Ord a) => [a] -> [a]
ordena [] = []
ordena (x:xs) =
    (ordena menores) ++ [x] ++ (ordena mayores)
  where menores = [a | a <- xs, a <= x]
        mayores = [b | b <- xs, b > x]
```

Recursión mutua: Par e impar

- ▶ Par e impar por recursión mutua:

```
par :: Int -> Bool
par 0 = True
par n = impar (n-1)
```

```
impar :: Int -> Bool
impar 0 = False
impar n = par (n-1)
```

- ▶ Cálculo:

impar 3		par 3
= par 2		= impar 2
= impar 1		= par 1
= par 0		= impar 0
= True		= False

Recursión mutua: Par e impar

- ▶ Par e impar por recursión mutua:

```
par :: Int -> Bool
par 0 = True
par n = impar (n-1)
```

```
impar :: Int -> Bool
impar 0 = False
impar n = par (n-1)
```

- ▶ Cálculo:

impar 3		par 3
= par 2		= impar 2
= impar 1		= par 1
= par 0		= impar 0
= True		= False

Recursión mutua: Par e impar

- ▶ Par e impar por recursión mutua:

```
par :: Int -> Bool
par 0 = True
par n = impar (n-1)
```

```
impar :: Int -> Bool
impar 0 = False
impar n = par (n-1)
```

- ▶ Cálculo:

impar 3		par 3
= par 2		= impar 2
= impar 1		= par 1
= par 0		= impar 0
= True		= False

Recursión mutua: Posiciones pares e impares

- ▶ (pares xs) son los elementos de xs que ocupan posiciones pares.
- ▶ (impares xs) son los elementos de xs que ocupan posiciones impares.

```
pares :: [a] -> [a]
pares []      = []
pares (x:xs) = x : impares xs
```

```
impares :: [a] -> [a]
impares []      = []
impares (_:xs) = pares xs
```

- ▶ Cálculo:

```
pares [1,3,5,7]
= 1:(impares [3,5,7])
= 1:(pares [5,7])
= 1:(5:(impares [7]))
= 1:(5:[])
= [1,5]
```


Recursión mutua: Posiciones pares e impares

- ▶ (pares xs) son los elementos de xs que ocupan posiciones pares.
- ▶ (impares xs) son los elementos de xs que ocupan posiciones impares.

```
pares :: [a] -> [a]
pares []      = []
pares (x:xs) = x : impares xs
```

```
impares :: [a] -> [a]
impares []      = []
impares (_:xs) = pares xs
```

- ▶ Cálculo:

```
pares [1,3,5,7]
= 1:(impares [3,5,7])
= 1:(pares [5,7])
= 1:(5:(impares [7]))
= 1:(5:[])
= [1,5]
```

Recursión mutua: Posiciones pares e impares

- ▶ (pares xs) son los elementos de xs que ocupan posiciones pares.
- ▶ (impares xs) son los elementos de xs que ocupan posiciones impares.

```
pares :: [a] -> [a]
pares []      = []
pares (x:xs) = x : impares xs
```

```
impares :: [a] -> [a]
impares []      = []
impares (_:xs) = pares xs
```

- ▶ Cálculo:

```

pares [1,3,5,7]
= 1:(impares [3,5,7])
= 1:(pares [5,7])
= 1:(5:(impares [7]))
= 1:(5:[])
= [1,5]
```

Aplicación del método: La función product

- ▶ Paso 1: Definir el tipo:

```
product :: [Int] -> Int
```

- ▶ Paso 2: Enumerar los casos:

```
product :: [Int] -> Int
product []      =
product (n:ns) =
```

- ▶ Paso 3: Definir los casos simples:

```
product :: [Int] -> Int
product []      = 1
product (n:ns) =
```

Aplicación del método: La función product

- ▶ Paso 1: Definir el tipo:

```
product :: [Int] -> Int
```

- ▶ Paso 2: Enumerar los casos:

```
product :: [Int] -> Int
product []      =
product (n:ns) =
```

- ▶ Paso 3: Definir los casos simples:

```
product :: [Int] -> Int
product []      = 1
product (n:ns) =
```

Aplicación del método: La función product

- ▶ Paso 1: Definir el tipo:

```
product :: [Int] -> Int
```

- ▶ Paso 2: Enumerar los casos:

```
product :: [Int] -> Int
product []      =
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- ▶ Paso 3: Definir los casos simples:

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Aplicación del método: La función product

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product []      =
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- ▶ Paso 3: Definir los casos simples:

```
product :: [Int] -> Int
product []      = 1
product (n:ns) =
```

Aplicación del método: La función product

- ▶ Paso 4: Definir los otros casos:

```
product :: [Int] -> Int
product []      = 1
product (n:ns) = n * product ns
```

- ▶ Paso 5: Generalizar y simplificar:

```
product :: Num a => [a] -> a
product = foldr (*) 1
```

donde (`foldr op e l`) pliega por la derecha la lista `l` colocando el operador `op` entre sus elementos y el elemento `e` al final. Por ejemplo,

```
foldr (+) 6 [2,3,5] ~> 2+(3+(5+6)) ~> 16
foldr (-) 6 [2,3,5] ~> 2-(3-(5-6)) ~> -2
```

Aplicación del método: La función product

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```
product :: [Int] -> Int
product []      = 1
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Aplicación del método: La función product

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<code>foldr (+) 6 [2,3,5]</code>	$\rightsquigarrow 2+(3+(5+6))$	$\rightsquigarrow 16$
<code>foldr (-) 6 [2,3,5]</code>	$\rightsquigarrow 2-(3-(5-6))$	$\rightsquigarrow -2$

Aplicación del método: La función drop

- ▶ Paso 1: Definir el tipo:

```
drop :: Int -> [a] -> [a]
```

- ▶ Paso 2: Enumerar los casos:

```
drop :: Int -> [a] -> [a]
drop 0 []      =
drop 0 (x:xs) =
drop n []      =
drop n (x:xs) =
```

Aplicación del método: La función drop

- ▶ Paso 1: Definir el tipo:

```
drop :: Int -> [a] -> [a]
```

- ▶ Paso 2: Enumerar los casos:

```
drop :: Int -> [a] -> [a]
drop 0 []      =
drop 0 (x:xs) =
drop n []      =
drop n (x:xs) =
```

Aplicación del método: La función drop

- ▶ Paso 1: Definir el tipo:

```
drop :: Int -> [a] -> [a]
```

- ▶ Paso 2: Enumerar los casos:

```
drop :: Int -> [a] -> [a]
drop 0 []      =
drop 0 (x:xs) =
drop n []      =
drop n (x:xs) =
```

Aplicación del método: La función drop

- ▶ Paso 3: Definir los casos simples:

```
drop :: Int -> [a] -> [a]
drop 0 []      = []
drop 0 (x:xs) = x:xs
drop n []      = []
drop n (x:xs) =
```

- ▶ Paso 4: Definir los otros casos:

```
drop :: Int -> [a] -> [a]
drop 0 []      = []
drop 0 (x:xs) = x:xs
drop n []      = []
drop n (x:xs) = drop n xs
```

Aplicación del método: La función drop

- ▶ Paso 3: Definir los casos simples:

```
drop :: Int -> [a] -> [a]
drop 0 []      = []
drop 0 (x:xs) = x:xs
drop n []      = []
drop n (x:xs) =
```

- ▶ Paso 4: Definir los otros casos:

```
drop :: Int -> [a] -> [a]
drop 0 []      = []
drop 0 (x:xs) = x:xs
drop n []      = []
drop n (x:xs) = drop n xs
```

Aplicación del método: La función drop

- ▶ Paso 3: Definir los casos simples:

```
drop :: Int -> [a] -> [a]
drop 0 []          = []
drop 0 (x:xs)     = x:xs
drop n []          = []
drop n (x:xs)     =
```

- ▶ Paso 4: Definir los otros casos:

```
drop :: Int -> [a] -> [a]
drop 0 []          = []
drop 0 (x:xs)     = x:xs
drop n []          = []
drop n (x:xs)     = drop n xs
```

Aplicación del método: La función drop

- ▶ Paso 5: Generalizar y simplificar:

```
drop :: Integral b => b -> [a] -> [a]
drop 0 xs      = xs
drop n []     = []
drop n (_:xs) = drop n xs
```

Aplicación del método: La función drop

- ▶ Paso 5: Generalizar y simplificar:

```
drop :: Integral b => b -> [a] -> [a]
```

```
drop 0 xs      = xs
```

```
drop n []     = []
```

```
drop n (_:xs) = drop n xs
```

Aplicación del método: La función `init`

- ▶ `init` elimina el último elemento de una lista no vacía.
- ▶ Paso 1: Definir el tipo:

```
init :: [a] -> [a]
```

- ▶ Paso 2: Enumerar los casos:

```
init :: [a] -> [a]  
init (x:xs) =
```

- ▶ Paso 3: Definir los casos simples:

```
init :: [a] -> [a]  
init (x:xs) | null xs    = []  
            | otherwise =
```

Aplicación del método: La función `init`

- ▶ `init` elimina el último elemento de una lista no vacía.
- ▶ Paso 1: Definir el tipo:

```
init :: [a] -> [a]
```

- ▶ Paso 2: Enumerar los casos:

```
init :: [a] -> [a]  
init (x:xs) =
```

- ▶ Paso 3: Definir los casos simples:

```
init :: [a] -> [a]  
init (x:xs) | null xs    = []  
            | otherwise =
```

Aplicación del método: La función `init`

- ▶ `init` elimina el último elemento de una lista no vacía.
- ▶ Paso 1: Definir el tipo:

```
init :: [a] -> [a]
```

- ▶ Paso 2: Enumerar los casos:

```
init :: [a] -> [a]  
init (x:xs) =
```

- ▶ Paso 3: Definir los casos simples:

```
init :: [a] -> [a]  
init (x:xs) | null xs    = []  
            | otherwise =
```

Aplicación del método: La función `init`

- ▶ `init` elimina el último elemento de una lista no vacía.
- ▶ Paso 1: Definir el tipo:

```
init :: [a] -> [a]
```

- ▶ Paso 2: Enumerar los casos:

```
init :: [a] -> [a]  
init (x:xs) =
```

- ▶ Paso 3: Definir los casos simples:

```
init :: [a] -> [a]  
init (x:xs) | null xs    = []  
            | otherwise =
```

Aplicación del método: La función `init`

- ▶ Paso 4: Definir los otros casos:

```
init :: [a] -> [a]
init (x:xs) | null xs    = []
             | otherwise = x : init xs
```

- ▶ Paso 5: Generalizar y simplificar:

```
init :: [a] -> [a]
init []    = []
init (x:xs) = x : init xs
```

Aplicación del método: La función `init`

- ▶ Paso 4: Definir los otros casos:

```
init :: [a] -> [a]
init (x:xs) | null xs    = []
             | otherwise = x : init xs
```

- ▶ Paso 5: Generalizar y simplificar:

```
init :: [a] -> [a]
init []    = []
init (x:xs) = x : init xs
```

Aplicación del método: La función `init`

- ▶ Paso 4: Definir los otros casos:

```
init :: [a] -> [a]
init (x:xs) | null xs    = []
             | otherwise = x : init xs
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- ▶ Paso 5: Generalizar y simplificar:

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init :: [a] -> [a]
init []    = []
init (x:xs) = x : init xs
```

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