Ejemplo de sistema basado en conocimiento: Macsyma

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Introducción

- **1957: GPS (General Problem Solver): Newell, Simon y Shaw**
- **1961: SAINT (Symbolic Automatic INTegrator): J. Slagle**
- **1963: SINT (Symbolic INTegrator): J. Moses**
- **1963: REDUCE: Hearn.**
- **1968: MACSYMA (MAC's SYmbolic Manipulation System): J. Moses**
- **1979: SMP: Wolfram.**
- **1979: muMATH: Stoutemeyer y Rich.**
- **1980: MAPLE.**
- **1988: MATHEMATICA: Wolfram.**
- **1988: DERIVE: Stoutemeyer y Rich.**
- **1996: Maxima 5.2: W. Schelter**

Cálculos numéricos

• Aritmética

```
(C1) 2+3;
(D1) 5
(C2) 7/6+9/10;
     31
(D2) --
     15
```

• Definición de variables

```
(C1) x;
(D1) X
(C2) x:3;
(D2) 3
(C3) x^2;
(D3) 9
(C4) kill(x);
(D4) DONE
(C5) x^2;
      2
(D5) X
```

• Operaciones múltiples

```
(C1) (a:2, b:3, a+b);
(D1) 5
```

Cálculos numéricos

• Precisión

• Aritmética entera:

```
(C1) 2^100;
(D1) 1267650600228229401496703205376
```

• Aritmética real:

```
(C1) sqrt(2);
(D1) SQRT(2)
(C2) float(%);
(D2) 1.4142135623730950B0
(C3) fpprec;
(D3) 16
(C4) fpprec:50$
(C5) bfloat(d1);
(D5) 1.4142135623730950488016887242096980785696718753772B0
```

• Sumas y productos generalizados

```
(C1) sum(x^n,n,1,5);
      5    4    3    2
(D1) X  + X  + X  + X  + X
(C2) product(x+n*y,n,1,4);
(D2) (Y + X) (2 Y + X) (3 Y + X) (4 Y + X)
```

Algebra

• Definición de funciones

```
(C1) area(base,altura) := (base*altura)/2;
                            BASE ALTURA
(D1) AREA(BASE, ALTURA) := -----------
                                 2
(C2) area(4,7);
(D2) 14
(C3) area(2+3,2*3);
(D3) 15
(C4) area(x,2*y);
(D4) X Y
```

• Polinomios

• Operaciones con polinomios:

```
(C1) (3+x)^2*(5*x-1)+8*(x^2-3*x+4)*(x-2);
                   2                         2
(D1) 8 (X - 2) (X  - 3 X + 4) + (X + 3)  (5 X - 1)
(C2) expand(%);
         3       2
(D2) 13 X  - 11 X  + 119 X - 73
```

Algebra

- **División de polinomios:**

```
(C1) a : 2*x^3-3*x^2-5*x-5$
(C2) b : x-2$
(C3) quotient(a,b);
                  2
(D3)              2 X  + X - 3
(C4) remainder(a,b);
(D4)                    - 11
(C5) b*d3+d4;
                          2
(D5)              (X - 2) (2 X  + X - 3) - 11
(C6) expand(%);
                  3      2
(D6)              2 X  - 3 X  - 5 X - 5
```

- **Descomposición factorial de polinomios:**

```
(C1) (x-2)^2*(x+3);
                    2
(D1)               (X - 2)  (X + 3)
(C2) expand(d1);
                3    2
(D2)           X  - X  - 8 X + 12
(C3) factor(d2);
                    2
(D3)               (X - 2)  (X + 3)
```

Algebra

```
(C4) factor(y^4-x^4);

                          2    2
(D4) (Y - X) (Y + X) (Y  + X )
(C5) factor(x^(2*n)+2*x^n+1);
       N     2
(D5) (X  + 1)
(C6) factor(10!);
      8  4  2
(D6) 2  3  5  7
```

- **Máximo común divisor y mínimo común múltiplo:**

```
(C1) p : x^2-9$
(C2) q : x^2+6*x+9$
(C3) gcd(p,q);
(D3) X + 3
(C4) lcm(p,q);
                      2
(D4) (X - 3) (X + 3)
```

- **Valor numérico de un polinomio:**

```
(C1) at(x^2+y^3,[x=1,y=2]);
(D1) 9
```

Algebra

• Fracciones algebraicas

• Simplificación de fracciones algebraicas:

```
(C1) (b^2-a^2)/(b+a)^2;
      2    2
     B  - A
(D1) --------
             2
     (B + A)
(C2) ratsimp(%);
     B - A
(D2) -----
     B + A
```

• Operaciones con fracciones algebraicas:

```
(C1) (1/x-1/y)*(1/x+1/y)*(1+(x+y)/(x-y))*(2+2*y/(x-y));
      1   1   1   1    2 Y         Y + X
(D1) (- - -) (- + -) (----- + 2) (----- + 1)
      X   Y   Y   X   X - Y       X - Y
(C2) ratsimp(%);
     4 Y + 4 X
(D2) ---------
      3      2
     Y  - X Y
```

Algebra

- **Valor numérico de una fracción algebraica:**

```
(C1) (3*a-2*b^2)/(3*a*b);
                2
     3 A - 2 B
(D1) ----------
       3 A B
(C2) at(%,[a=-2,b=3]);
     4
(D2) -
     3
```

- **Verdadero valor de una fracción:**

```
(C1) (x^2-1)/(x-1);
      2
     X  - 1
(D1) ------
     X - 1
(C2) at(%,x=1);
Division by 0
(C3) at(ratsimp(d1),x=1);
(D3) 2
```

Algebra

- **Ecuaciones y sistemas**
  - **Ecuaciones:**

```
(C1) e1:x^4-(a^2+b^2)*x^2+a^2*b^2=0;
      4     2    2   2    2  2
(D1) X  - (B  + A ) X  + A  B  = 0
(C2) solve(e1,x);
(D2) [X = - A, X = A, X = - B, X = B]
(C3) e2:sqrt(3*x-2)-4=0;
(D3) SQRT(3 X - 2) - 4 = 0
(C4) solve(e2);
(D4) [X = 6]
(C5) e3:x^3-2*x^2+x-2=0;
      3      2
(D5) X  - 2 X  + X - 2 = 0
(C6) solve(e3);
(D6) [X = - %I, X = %I, X = 2]
(C7) solve(sin(x)*cos(x)=0,x);

SOLVE is using arc-trig functions to get a solution.
Some solutions will be lost.
                  %PI
(D7) [X = 0, X = ---]
                   2
```

Algebra

- **Sistemas:**

```
(C1) e1:x+y=8$
(C2) e2:x*y=12$
(C3) solve([e1,e2],[x,y]);
(D3) [[X = 2, Y = 6], [X = 6, Y = 2]]
(C4) solve([x^2=5,x=5],x);
(D4) []
```

Algebra lineal

• Definición de matrices

• Matrices generales:

```
(C1) a:matrix([4,0,6],[1,6,2],[3,9,0]);
     [ 4  0  6 ]
     [         ]
(D1) [ 1  6  2 ]
     [         ]
     [ 3  9  0 ]
(C2) a[3,2];
(D2) 9
(C3) row(a,3);
(D3) [ 3  9  0 ]
(C4) col(a,2);
     [ 0 ]
     [   ]
(D4) [ 6 ]
     [   ]
     [ 9 ]
```

Algebra lineal

- **Matrices especiales:**

```
(C1) zeromatrix(2,3);
     [ 0  0  0 ]
(D1) [         ]
     [ 0  0  0 ]
(C2) ident(2);
     [ 1  0 ]
(D2) [      ]
     [ 0  1 ]
(C3) diagmatrix(2,a);
     [ A  0 ]
(D3) [      ]
     [ 0  A ]
```

- **Definición de matrices mediante funciones:**

```
(C1) f[i,j] := 10*i+j$
(C2) m : genmatrix(f,2,3)$
(C3) m;
     [ 11  12  13 ]
(D3) [            ]
     [ 21  22  23 ]
```

Algebra lineal

- **Definición reducida de matrices:**

```
(C1) a:entermatrix(3,3);

Is the matrix  1. Diagonal  2. Symmetric  3. Antisymmetric
               4. General
Answer 1, 2, 3 or 4 : 1;
Row 1 Column 1:  x;
Row 2 Column 2:  y;
Row 3 Column 3:  z;

Matrix entered.
     [ X  0  0 ]
     [         ]
(D1) [ 0  Y  0 ]
     [         ]
     [ 0  0  Z ]
(C2) a;
     [ X  0  0 ]
     [         ]
(D2) [ 0  Y  0 ]
     [         ]
     [ 0  0  Z ]
```

Algebra lineal

- **Operaciones con matrices:**

```
(C1) A : matrix([9,8],[7,6]);
     [ 9  8 ]
(D1) [      ]
     [ 7  6 ]
(C2) B : matrix([0,1],[3,2]);
     [ 0  1 ]
(D2) [      ]
     [ 3  2 ]
(C3) A + B;
     [ 9   9 ]
(D3) [       ]
     [ 10  8 ]
(C4) A - B;
     [ 9  7 ]
(D4) [      ]
     [ 4  4 ]
(C5) A . B;
     [ 24  25 ]
(D5) [        ]
     [ 18  19 ]
```

Algebra lineal

- **Matriz traspuesta:**

```
(C1) A : matrix([5,3,6],[2,0,7],[8,1,0])$
(C2) transpose(a);
     [ 5  2  8 ]
     [         ]
(D2) [ 3  0  1 ]
     [         ]
     [ 6  7  0 ]
```

- **Matriz inversa:**

```
(C1) A : matrix([-4,9],[1,-2])$
(C2) invert(A);
     [ 2  9 ]
(D2) [      ]
     [ 1  4 ]
(C3) A . %;
     [ 1  0 ]
(D3) [      ]
     [ 0  1 ]
```

Algebra lineal

- **Potencia de matrices:**

```
(C1) A : matrix([1,1],[0,1]);
     [ 1  1 ]
(D1) [      ]
     [ 0  1 ]
(C2) A^^2;
     [ 1  2 ]
(D2) [      ]
     [ 0  1 ]
(C3) A^^3;
     [ 1  3 ]
(D3) [      ]
     [ 0  1 ]
(C4) A^^-1;
     [ 1  - 1 ]
(D4) [        ]
     [ 0   1  ]
(C5) invert(A);
     [ 1  - 1 ]
(D5) [        ]
     [ 0   1  ]
```

Algebra lineal

- **Determinante:**

```
(C1) A : matrix([1,1,1],[x,y,z],[y+z,z+x,x+y]);
     [   1      1      1   ]
     [                     ]
(D1) [   X      Y      Z   ]
     [                     ]
     [ Z + Y  Z + X  Y + X ]
(C2) determinant(A);
(D2) Z(Z+Y)-Y(Z+Y)-Z(Z+X)+X(Z+X)+Y(Y+X)-X(Y+X)
(C3) expand(%);
(D3) 0
```

- **Rango:**

```
(C1) A : matrix([-1,1,-2],[1,1,0],[2,1,1]);
     [ - 1  1  - 2 ]
     [             ]
(D1) [  1   1   0  ]
     [             ]
     [  2   1   1  ]
(C2) rank(A);
(D2) 2
```

Cálculo

- **Límites:**

```
(C1) limit((x^2-4)/(x-2),x,2);
(D1) 4
(C2) limit((x^n-a^n)/(x-a),x,a);
      N - 1
(D2) A      N
(C3) limit((x^(1/n)-1)/(x^(1/m)-1),x,1);
     M
(D3) -
     N
(C4) limit(1/(x-1)^4,x,1);
(D4) INF
(C5) limit(-1/(x-1)^4,x,1);
(D5) MINF
(C6) limit((4*x+3)/(2*x-1),x,inf);
(D6) 2
(C7) limit((1+2/x)^x,x,inf);
       2
(D7) %E
(C8) limit((cos(x))^(1/x),x,0);
(D8) 1
```

Cálculo

- **Límites laterales:**

```
(C9) f(x) := (2*x+abs(x))/(4*x-3*abs(x));
               2 X + ABS(X)
(D9) F(X) := --------------
              4 X - 3 ABS(X)
(C10) limit(f(x),x,0);
                   ABS(X)             2 X
(D10) limit  -------------- + --------------
      X -> 0 4 X - 3 ABS(X)   4 X - 3 ABS(X)
(C11) limit(f(x),x,0,minus);
      1
(D11) -
      7
(C12) limit(f(x),x,0,plus);
(D12) 3
```

Cálculo

- **Derivadas:**

```
(C1) diff((sin(4*x))^3,x);
                     2
(D1) 12 COS(4 X) SIN (4 X)
(C2) diff(x^4+3*x^2-5,x,2);
         2
(D2) 12 X  + 6
(C3) diff(x^y,x);
      Y - 1
(D3) X      Y
(C4) diff(x^y,y);
      Y
(D4) X  LOG(X)
```

- **Fórmula de Taylor:**

```
(C1) taylor(%e^x,x,0,5);
                  2    3    4    5
                 X    X    X    X
(D1)/T/ 1 + X + -- + -- + -- + --- + . . .
                 2    6    24   120

(C2) taylor(x^3-4*x^2+7,x,1,3);
                       2         3
(D2)/T/ 4 - 5(X-1)-(X  - 1)+(X - 1)  + . . .
```

Cálculo

- **Integrales indefinidas:**

```
(C1) integrate(1/x,x);
(D1) LOG(X)
(C2) integrate(1/(x*log(x)),x);
(D2) LOG(LOG(X))
(C3) integrate(sin(3*x)*(cos(3*x))^4,x);
           5
       COS (3 X)
(D3) - ---------
           15
(C4) integrate(x*%e^x,x);
                 X
(D4) (X - 1) %E
(C5) integrate(1/(x^2+a^2),x);
           X
     ATAN(-)
           A
(D5) -------
        A
```

Cálculo

- **Integrales definidas:**

```
(C1) integrate(3*x^2,x,1,2);
(D1) 7
(C2) f(x) := integrate(1/log(t),t,1,x)$
(C3) f(y);
Is  Y - 1  positive, negative, or zero?
pos;
      Y
     /
     [    1
(D3) I  ------ dT
     ]  LOG(T)
     /
      1
(C4) diff(f(x),x);
Is  X - 1  positive, negative, or zero?
pos;
       1
(D4) ------
     LOG(X)
(C5) integrate(1/x^2,x,-1,1);
Integral is divergent
(C6) integrate(1/(1+x^2),x,0,inf);
     %PI
(D6) ---
      2
```

Cálculo

- **Integración numérica:**

```
(C1) showtime : true$
Evaluation took 0.00 seconds (0.64 elapsed)
(C2) integrate(1/x,x,1,2);
Evaluation took 0.68 seconds (3.08 elapsed)
(D2) LOG(2)
(C3) float(d2);
Evaluation took 0.00 seconds (00.70 elapsed)
(D3) 0.69314718055994529
(C4) romberg(1/x,x,1,2);
Evaluation took 00.10 seconds (00.20 elapsed)
(D4) 0.69314790148123473
(C5) integrate(1/log(x),x,2,4);
Evaluation took 0.45 seconds (1.09 elapsed)
      4
     /
     [    1
(D5) I  ------ dX
     ]  LOG(X)
     /
      2
(C6) romberg(1/log(x),x,2,4);
Evaluation took 00.30 seconds (0.30 elapsed)
(D6) 1.9224216144693915
```

Cálculo

- **Sumatorios:**

```
(C1) sum(i^2,i,1,n);

     ====
     \       2
(D1)  >     I
     /
     ====
     I = 1
(C2) simpsum:true;
(D2) TRUE
(C3) sum(i^2,i,1,n);
        3      2
     2 N  + 3 N  + N
(D3) ---------------
             6

(C4) sum(x^k,k,0,n);
      N + 1
     X      - 1
(D4) ----------
       X - 1
```

Cálculo

- **Series numéricas:**

```
(C1) sum((-1/4)^n,n,1,inf);
       1
(D1) - -
       5
(C2) sum(r^n,n,0,inf);
Is  ABS(R) - 1  positive, negative, or zero?

neg;
       1
(D2) -----
     1 - R
(C3) sum(1/n,n,1,inf);
(D3) INF
(C4) sum((-1)^n,n,1,inf);
(D4) UNDEFINED
```

Cálculo

- **Ecuaciones diferenciales:**

```
(C1) e1 : 'diff(f(x),x)=3*x^2+2*x;
     d               2
(D1) -- (F(X)) = 3 X  + 2 X
     dX
(C2) desolve(e1,f(x));
              3    2
(D2) F(X) = X  + X  + F(0)
(C3) e2 : 'diff(y(x),x)=a*y(x);
     d
(D3) -- (Y(X)) = A Y(X)
     dX
(C4) desolve(e1,y(x));
                    A X
(D4) Y(X) = Y(0) %E
(C5) atvalue(y(x),x=0,1)$
(C6) desolve(e,y(x));
               A X
(D6) Y(X) = %E
```

Programación

- **Iteración:**

```
(C1) for i from 1 thru 3 do print("I =",i);
I = 1
I = 2
I = 3
(D1) DONE
(C2) for i from 1 step 2 thru 3 do print("I =",i);
I = 1
I = 3
(D2) DONE
(C3) i:1$
(C4) for i while i<=2 do print("I =",i);
I = 1
I = 2
(D4) DONE
(C5) i;
(D5) 1
(C6) for i unless i>2 do print("I =",i);
I = 1
I = 2
(D6) DONE
(C7)  for i in [1,2] do print("I =",i);
I = 1
I = 2
(D7) DONE
```

Programación

- **Condicionales:**

```
(C1) fib(n) := if n=1 or n=2
                  then 1
                  else fib(n-1)+fib(n-2)$
(C2) fib(6);
(D2) 8
```

- **Definiciones recursivas mediante índices:**

```
(C1) fact[0] : 1$
(C2) fact[n] := n*fact[n-1]$
(C3) fact[3];
(D3) 6
(C4) arrayinfo(fact);
(D4) [HASHED, 1, [0], [1], [2], [3]]
```

- **Booleanos:**

```
(C1) is(9 > 2+3);
(D1) TRUE
(C2) is(9 > 2+3 and 2^3 = 2*3);
(D2) FALSE
(C3) is(9 > 2+3 or 2^3 = 2*3);
(D3) TRUE
(C4) x:9$
(C5) is(x=8+1);
(D5) TRUE
```

Programación

- **Bloques y listas:**

```
(C1) es_primo(n) := is(factor(n) = n)$
(C2) es_primo(7);
(D2) TRUE
(C3) es_primo(6);
(D3) FALSE
(C4) primos(n,m) :=
      for i from n thru m
          do if es_primo(i) then print(i)$
(C5) primos(10,30);
11
13
17
19
23
29
(D5) DONE
```

Programación

```
(C6) lista_de_primos(n,m) :=
       (res:[],
        for i from n thru m
            do if es_primo(i)
                  then res:cons(i,res),
        reverse(res))$
(C7) lista_de_primos(10,30);
(D7) [11, 13, 17, 19, 23, 29]
(C8) res;
(D8) [29, 23, 19, 17, 13, 11]
(C9) kill(res);
(D9) DONE
(C10) lista_de_primos(n,m) := block([res],
        res:[],
        for i from n thru m
            do if es_primo(i)
                  then res:cons(i,res),
        reverse(res))$
(C11) lista_de_primos(10,30);
(D11) [11, 13, 17, 19, 23, 29]
(C12) res;
(D12) RES
```

Programación

- **Método de Newton:**

$$f(x) = 0$$

$$x_0 = a, x_n = x_{n-1} - \frac{f(x_{n-1})}{f'(x_{n-1})}$$

```
(C1) newton(f,aprox) := block([numer],
      numer:true,
      do (if abs(f(aprox)) < 0.001 then return(aprox),
          aprox:at(x-f(x)/diff(f(x),x),x=aprox))$
(C2) g(x) := x^2-3$
(C3) newton(g,10);
(D3) 1.7320982711195376
(C4) h(x) := 2*x-log(x)-4$
(C5) newton(h,10);
(D5) 2.4475434805898919
(C6) h(d5);
(D6) 2.1006076442731114E-6
```

Bibliografía

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- [Davenport–88]
  J.H. Davenport, Y. Siret y E. Tournier "Computer Algebra (Systems and algoritms for algebraic computation)" (Academic Press, 1988)