Tema 8: Razonamiento automático y programación

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El factorial mediante resolución

- **Entrada**

```
set(prolog_style_variables).
set(binary_res).

list(usable).
fact(0,1).
-fact((X - 1),Y) | fact(X,(X * Y)).

-p(fact(X,Y)) | -fact(X,Y) | $ans(Y).
end_of_list.

list(demodulators).
1-1 = 0.   2-1 = 1.   3-1 = 2.  4-1 = 3.
end_of_list.

list(sos).
p(fact(4,Y)).
end_of_list.
```

- **Demostración**

```
1 [] fact(0,1).
2 [] -fact(X-1,Y)|fact(X,X*Y).
3 [] -p(fact(X,Y))| -fact(X,Y)|$ans(Y).
4 [] 1-1=0.
5 [] 2-1=1.
6 [] 3-1=2.
7 [] 4-1=3.
8 [] p(fact(4,Y)).
9 [binary,8.1,3.1] -fact(4,A)|$ans(A).
10 [binary,9.1,2.2,demod,7] $ans(4*A)| -fact(3,A).
11 [binary,10.1,2.2,demod,6] $ans(4*3*A)| -fact(2,A).
12 [binary,11.1,2.2,demod,5] $ans(4*3*2*A)| -fact(1,A).
13 [binary,12.1,2.2,demod,4] $ans(4*3*2*1*A)| -fact(0,A).
14 [binary,13.1,1.1] $ans(4*3*2*1*1).
```

El factorial con producto evaluable

- **Entrada**

```
set(prolog_style_variables).
set(binary_res).

list(usable).
fact(0,1).
-fact((X - 1),Y) | fact(X,$PROD(X,Y)).
-p(fact(X,Y)) | -fact(X,Y) | resp(Y).
end_of_list.

list(demodulators).
1-1 = 0.   2-1 = 1.   3-1 = 2.  4-1 = 3.
end_of_list.

list(sos).
p(fact(4,Y)).
end_of_list.

list(passive).
-resp(X) | $ans(X).
end_of_list.
```

El factorial con producto evaluable

- **Prueba**

```
1 [] fact(0,1).
2 [] -fact(X-1,Y)|fact(X,$PROD(X,Y)).
3 [] -p(fact(X,Y))| -fact(X,Y)|resp(Y).
4 [] 1-1=0.
5 [] 2-1=1.
6 [] 3-1=2.
7 [] 4-1=3.
8 [] p(fact(4,Y)).
9 [] -resp(Y)|$ans(Y).
10 [binary,8.1,3.1]
   -fact(4,A)|resp(A).
11 [binary,10.1,2.2,demod,7]
   resp($PROD(4,A))| -fact(3,A).
12 [binary,11.2,2.2,demod,6]
   resp($PROD(4,$PROD(3,A)))| -fact(2,A).
13 [binary,12.2,2.2,demod,5]
   resp($PROD(4,$PROD(3,$PROD(2,A))))| -fact(1,A).
14 [binary,13.2,2.2,demod,4]
   resp($PROD(4,$PROD(3,$PROD(2,$PROD(1,A)))))| -fact(0,A)
16 [binary,14.2,1.1,demod]
   resp(24).
17 [binary,16.1,9.1]
   $ans(24).
```

El factorial con producto evaluable (II)

```
set(prolog_style_variables).
set(binary_res).
make_evaluable(_*_, $PROD(_,_)).

list(usable).
fact(0,1).
-fact((X - 1),Y) | fact(X,$PROD(X,Y)).
-p(fact(X,Y)) | -fact(X,Y) | resp(Y).
end_of_list.

list(demodulators).
1-1 = 0.   2-1 = 1.   3-1 = 2.  4-1 = 3.
end_of_list.

list(sos).
p(fact(4,Y)).
end_of_list.

list(passive).
-resp(X) | $ans(X).
end_of_list.
```

El factorial con producto evaluable (II)

- **Prueba**

```
1 [] fact(0,1).
2 [] -fact(X-1,Y)|fact(X,X*Y).
3 [] -p(fact(X,Y))| -fact(X,Y)|resp(Y).
4 [] 1-1=0.
5 [] 2-1=1.
6 [] 3-1=2.
7 [] 4-1=3.
8 [] p(fact(4,Y)).
9 [] -resp(X)|$ans(X).
10 [binary,8.1,3.1] -fact(4,A)|resp(A).
11 [binary,10.1,2.2,demod,7] resp(4*A)| -fact(3,A).
12 [binary,11.2,2.2,demod,6] resp(4*3*A)| -fact(2,A).
13 [binary,12.2,2.2,demod,5] resp(4*3*2*A)| -fact(1,A).
14 [binary,13.2,2.2,demod,4] resp(4*3*2*1*A)| -fact(0,A).
16 [binary,14.2,1.1,demod] resp(24).
17 [binary,16.1,9.1] $ans(24).
```

Con producto y resta evaluables

- **Entrada**

```
set(prolog_style_variables).
make_evaluable(_*_, $PROD(_,_)).
make_evaluable(_-_, $DIFF(_,_)).
set(binary_res).

list(usable).
fact(0,1).
-fact(X-1,Y) | fact(X,X*Y).
-p(fact(X,Y)) | -fact(X,Y) | resp(Y).
end_of_list.

list(sos).
p(fact(4,Y)).
end_of_list.

list(passive).
-resp(X) | $ans(X).
end_of_list.
```

- **Prueba**

```
1 [] fact(0,1).
2 [] -fact(X-1,Y)|fact(X,X*Y).
3 [] -p(fact(X,Y))| -fact(X,Y)|resp(Y).
4 [] p(fact(4,Y)).
5 [] -resp(X)|$ans(X).
6 [binary,4.1,3.1] -fact(4,A)|resp(A).
7 [binary,6.1,2.2,demod] resp(4*A)| -fact(3,A).
8 [binary,7.2,2.2,demod] resp(4*3*A)| -fact(2,A).
9 [binary,8.2,2.2,demod] resp(4*3*2*A)| -fact(1,A).
10 [binary,9.2,2.2,demod] resp(4*3*2*1*A)| -fact(0,A).
12 [binary,10.2,1.1,demod] resp(24).
13 [binary,12.1,5.1] $ans(24).
```

El factorial mediante demodulación

- **Entrada**

```
set(prolog_style_variables).
make_evaluable(_*_, $PROD(_,_)).
make_evaluable(_-_, $DIFF(_,_)).
make_evaluable(_>_, $GT(_,_)).
set(binary_res).

list(demodulators).
fact(0) = 1.
X>0 -> fact(X) = X*fact(X-1).
end_of_list.

list(usable).
-p(X) | $ans(factorial,X,fact(X)).
end_of_list.

list(sos).
p(4).
end_of_list.
```

- **Prueba**

```
1 [] fact(0)=1.
2 [] X>0->fact(X)=X*fact(X-1).
3 [] -p(X)|$ans(factorial,X,fact(X)).
4 [] p(4).
5 [binary,4.1,3.1,demod,2,2,2,2,1] $ans(factorial,4,24).
```

El factorial mediante IF

- **Entrada**

```
set(prolog_style_variables).
make_evaluable(_*_, $PROD(_,_)).
make_evaluable(_-_, $DIFF(_,_)).
make_evaluable(_==_, $EQ(_,_)).
set(binary_res).

list(demodulators).
fact(X) = $IF(X==0, 1, X*fact(X-1)).
end_of_list.

list(usable).
-p(X) | $ans(factorial,X,fact(X)).
end_of_list.

list(sos).
p(4).
end_of_list.
```

- **Prueba**

```
1 [] fact(X)=$IF(X==0,1,X*fact(X-1)).
2 [] -p(X)|$ans(factorial,X,fact(X)).
3 [] p(5).
4 [binary,3.1,2.1,demod,1,1,1,1,1] $ans(factorial,4,24).
```

Máximo común divisor

- **Entrada**

```
set(prolog_style_variables).
set(binary_res).
make_evaluable(_-_, $DIFF(_,_)).
make_evaluable(_<_, $LT(_,_)).
assign(max_proofs,-1).

list(demodulators).
mcd(0,X) = X.
mcd(X,0) = X.
mcd(X,X) = X.
X<Y -> mcd(X,Y) = mcd(X,Y-X).
Y<X -> mcd(X,Y) = mcd(X-Y,Y).
end_of_list.

list(usable).
-p(X,Y) | $ans(mcd,X,Y,mcd(X,Y)).
end_of_list.

list(sos).
p(12,15).    p(12,13).    p(2,0).
end_of_list.
```

Máximo común divisor

- **Salida**

```
---------------- PROOF ----------------
3 [] mcd(X,X)=X.
4 [] X<Y->mcd(X,Y)=mcd(X,Y-X).
5 [] Y<X->mcd(X,Y)=mcd(X-Y,Y).
6 [] -p(X,Y)|$ans(mcd,X,Y,mcd(X,Y)).
7 [] p(12,15).
10 [binary,7.1,6.1,demod,4,5,5,5,3]
   $ans(mcd,12,15,3).

---------------- PROOF ----------------
3 [] mcd(X,X)=X.
4 [] X<Y->mcd(X,Y)=mcd(X,Y-X).
5 [] Y<X->mcd(X,Y)=mcd(X-Y,Y).
6 [] -p(X,Y)|$ans(mcd,X,Y,mcd(X,Y)).
8 [] p(12,13).
11 [binary,8.1,6.1,demod,4,5,5,5,5,5,5,5,5,5,5,5,3]
   $ans(mcd,12,13,1).

---------------- PROOF ----------------
2 [] mcd(X,0)=X.
6 [] -p(X,Y)|$ans(mcd,X,Y,mcd(X,Y)).
9 [] p(2,0).
12 [binary,9.1,6.1,demod,2]
   $ans(mcd,2,0,2).
```

Máximo común divisor

- **Entrada**

```
set(prolog_style_variables).
make_evaluable(_-_, $DIFF(_,_)).
make_evaluable(_<_, $LT(_,_)).
make_evaluable(_==_, $EQ(_,_)).
set(binary_res).

list(demodulators).
mcd(X,Y) = $IF(X==0, Y,
                     $IF(Y==0, X,
                               $IF(X<Y, mcd(X,Y-X),
                                        mcd(Y,X-Y)))).
end_of_list.

list(usable).
-p(X,Y) | $ans(mcd,X,Y,mcd(X,Y)).
end_of_list.

list(sos).
p(12,15).   p(12,7).   p(2,0).
end_of_list.
```

Máximo común divisor

- **Salida**

```
1 [] mcd(X,Y)=$IF(X==0,Y,
                  $IF(Y==0,X,
                      $IF(X<Y,mcd(X,Y-X),
                          mcd(Y,X-Y)))).
2 [] -p(X,Y)|$ans(mcd,X,Y,mcd(X,Y)).
3 [] p(12,15).
4 [] p(12,7).
5 [] p(2,0).

-> EMPTY CLAUSE 6 [binary,3.1,2.1,demod,1,1,1,1,1,1]
   $ans(mcd,12,15,3).
-> EMPTY CLAUSE 7 [binary,4.1,2.1,demod,1,1,1,1,1,1,1]
   $ans(mcd,12,7,1).
-> EMPTY CLAUSE 8 [binary,5.1,2.1,demod,1]
   $ans(mcd,2,0,2).
```

Listas: La relación de pertenencia

- **Entrada**

```
set(prolog_style_variables).
set(binary_res).
assign(max_proofs,-1).

list(demodulators).
pertenece(X,[]) = $F.
$ID(X,Y) -> pertenece(X,[Y|L]) = $T.
$LNE(X,Y) -> pertenece(X,[Y|L]) = pertenece(X,L).
end_of_list.

list(usable).
-p(pertenece(X,Y)) | -pertenece(X,Y) | $ans(pert,X,Y).
-p(pertenece(X,Y)) |  pertenece(X,Y) | $ans(no_pert,X,Y).
end_of_list.

list(sos).
p(pertenece(a,[])).          p(pertenece(a,[a,b,c])).
p(pertenece(b,[a,b,c])).     p(pertenece(d,[a,b,c])).
end_of_list.
```

- **Salida**

```
10 [binary,6.1,5.1,demod,1]
   $ans(no_pert,a,[]).

12 [binary,7.1,4.1,demod,2]
   $ans(pert,a,[a,b,c]).

14 [binary,8.1,4.1,demod,3,2]
   $ans(pert,b,[a,b,c]).

15 [binary,9.1,5.1,demod,3,3,3,1]
   $ans(no_pert,d,[a,b,c]).
```

Listas: La relación de pertenencia

- **Entrada**

```
set(prolog_style_variables).
set(binary_res).
assign(max_proofs,-1).

list(demodulators).
pertenece(X,[])    = $F.
pertenece(X,[Y|L]) = $IF($ID(X,Y), $T,
                                   pertenece(X,L)).
end_of_list.

list(usable).
-p(pertenece(X,Y)) | -pertenece(X,Y) | $ans(pert,X,Y).
-p(pertenece(X,Y)) |  pertenece(X,Y) | $ans(no_pert,X,Y).
end_of_list.

list(sos).
p(pertenece(a,[])).          p(pertenece(a,[a,b,c])).
p(pertenece(b,[a,b,c])).     p(pertenece(d,[a,b,c])).
end_of_list.
```

- **Salida**

```
9 [binary,5.1,4.1,demod,1] $ans(no_pert,a,[]).
10 [binary,6.1,3.1,demod,2] $ans(pert,a,[a,b,c]).
11 [binary,7.1,3.1,demod,2,2] $ans(pert,b,[a,b,c]).
12 [binary,8.1,4.1,demod,2,2,2,1] $ans(no_pert,d,[a,b,c]).
```

Concatenación de listas

- **Entrada**

```
set(prolog_style_variables).
set(binary_res).
assign(max_proofs,-1).

list(demodulators).
concatenacion([],L)      = L.
concatenacion([X|L1],L2) = [X|concatenacion(L1,L2)].
end_of_list.

list(usable).
-p(concatenacion(X,Y))
 | $ans(concatenacion,X,Y,concatenacion(X,Y)).
end_of_list.

list(sos).
p(concatenacion([b,e],[a,b,c,d])).
end_of_list.
```

- **Salida**

```
1 [] concatenacion([],L)=L.
2 [] concatenacion([X|L1],L2)=[X|concatenacion(L1,L2)].
3 [] -p(concatenacion(X,Y))
     |$ans(concatenacion,X,Y,concatenacion(X,Y)).
4 [] p(concatenacion([b,e],[a,b,c,d])).
5 [binary,4.1,3.1,demod,2,2,1]
     $ans(concatenacion,[b,e],[a,b,c,d],[b,e,a,b,c,d]).
```

Inversión de listas

- **Entrada**

```
set(prolog_style_variables).
set(binary_res).
assign(max_proofs,-1).

list(demodulators).
inversa(L)              = inversa_aux(L,[]).
inversa_aux([], L)      = L.
inversa_aux([X|L1],L2) = inversa_aux(L1,[X|L2]).
end_of_list.

list(usable).
-p(inversa(X)) | $ans(inversa,X,inversa(X)).
end_of_list.

list(sos).
p(inversa([a,b,c])).
end_of_list.
```

- **Salida**

```
1 [] inversa(L)=inversa_aux(L,[]).
2 [] inversa_aux([],L)=L.
3 [] inversa_aux([X|L1],L2)=inversa_aux(L1,[X|L2]).
4 [] -p(inversa(X))|$ans(inversa,X,inversa(X)).
5 [] p(inversa([a,b,c])).
6 [binary,5.1,4.1,demod,1,3,3,3,2]
  $ans(inversa,[a,b,c],[c,b,a]).
```

Operaciones conjuntistas

- **Entrada**

```
set(prolog_style_variables).
make_evaluable(_&_, $AND(_,_)).
set(binary_res).
assign(max_proofs,-1).

list(demodulators).
pertenece(X,[])    = $F.
pertenece(X,[Y|L]) =
   $IF($ID(X,Y), $T,
                 pertenece(X,L)).

subconjunto([],L)      = $T.
subconjunto([X|L1],L2) =
   (pertenece(X,L2) & subconjunto(L1,L2)).

interseccion([],L)      = [].
interseccion([X|L1],L2) =
   $IF(pertenece(X,L2), [X|interseccion(L1,L2)],
                        interseccion(L1,L2)).

union([],L)      = L.
union([X|L1],L2) =
   $IF(pertenece(X,L2), union(L1,L),
                        [X|union(L1,L2)]).
end_of_list.
```

Operaciones conjuntistas

```
list(usable).
-p(subconjunto(X,Y))  | -subconjunto(X,Y)
 | $ans(subc,X,Y).
-p(subconjunto(X,Y))  | subconjunto(X,Y)
 | $ans(no_subc,X,Y).
-p(interseccion(X,Y))
 | $ans(interseccion,X,Y,interseccion(X,Y)).
-p(union(X,Y)) | $ans(union,X,Y,union(X,Y)).
end_of_list.

list(sos).
p(subconjunto([],[a,b])).
p(subconjunto([a],[a,b])).
p(subconjunto([a,b],[a,b])).
p(subconjunto([c],[a,b])).
p(subconjunto([a,c],[a,b])).
p(interseccion([b,d],[a,b,c,d])).
p(union([b,e],[a,b,c,d])).
end_of_list.
```

- **Salida**

```
12 [binary,7.1,5.1,demod,3]
   $ans(subc,[],[a,b]).
13 [binary,8.1,5.1,demod,4,2,3]
   $ans(subc,[a],[a,b]).
14 [binary,10.1,6.1,demod,4,2,2,1,3]
   $ans(no_subc,[c],[a,b]).
15 [binary,9.1,5.1,demod,4,2,4,2,2,3]
   $ans(subc,[a,b],[a,b]).
16 [binary,11.1,6.1,demod,4,2,4,2,2,1,3]
   $ans(no_subc,[a,c],[a,b]).
25 [binary,18.1,11.1,demod,6,2,2,6,2,2,2,2,5]
   $ans(interseccion,[b,d],[a,b,c,d],[b,d]).
26 [binary,19.1,12.1,demod,8,2,2,8,2,2,2,2,1,7]
   $ans(union,[b,e],[a,b,c,d],[e,a,b,c,d]).
```

Ordenación

- **Entrada**

```
set(prolog_style_variables).
make_evaluable(_@<=_, $LE(_,_)).
make_evaluable(_@>_,  $GT(_,_)).
set(binary_res).
assign(max_proofs,-1).

list(demodulators).
concatenacion([],L)      = L.
concatenacion([X|L1],L2) = [X|concatenacion(L1,L2)].

ordenacion([])    = [].
ordenacion([X|L]) =
   concatenacion(ordenacion(menores(X,L)),
                            [X|ordenacion(mayores(X,L))]).

menores(X,[])    = [].
menores(X,[Y|L]) = $IF(Y @<= X, [Y|menores(X,L)],
                                 menores(X,L)).

mayores(X,[])    = [].
mayores(X,[Y|L]) = $IF(Y @> X, [Y|mayores(X,L)],
                               mayores(X,L)).
end_of_list.

list(usable).
-p(ordenacion(X)) | $ans(ordenacion,X,ordenacion(X)).
end_of_list.

list(sos).
p(ordenacion([3,1,2])).
end_of_list.
```

Ordenación

- **Salida**

```
---------------- PROOF ----------------
1 [] concatenacion([],L)=L.
2 [] concatenacion([X|L1],L2)=
     [X|concatenacion(L1,L2)].
3 [] ordenacion([])=[].
4 [] ordenacion([X|L])=
     concatenacion(ordenacion(menores(X,L)),
                   [X|ordenacion(mayores(X,L))]).
5 [] menores(X,[])=[].
6 [] menores(X,[Y|L])=
     $IF(Y@<=X,[Y|menores(X,L)],menores(X,L)).
7 [] mayores(X,[])=[].
8 [] mayores(X,[Y|L])=
     $IF(Y@>X,[Y|mayores(X,L)],mayores(X,L)).
9 [] -p(ordenacion(X))|$ans(ordenacion,X,ordenacion(X)).
10 [] p(ordenacion([3,1,2])).
11 [binary,10.1,9.1,
    demod,4,6,6,5,4,6,5,3,8,7,4,5,3,7,3,1,1,8,8,7,3,2,2,1]
   $ans(ordenacion,[3,1,2],[1,2,3]).
------------ end of proof -------------
```